

Approximants de Padé et TI 92

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28 juin 2002

1 Ecriture d'une procédure déterminant les coefficients d'un polynôme donné

Soit P un polynôme de la variable x de degré n . Si $P(x) = a_0 + a_1x + a_2x^2 + \dots + a_nx^n$ alors pour tout entier k inférieur ou égal à n ,

$$P^{(k)}(x) = k!a_k + (k+1)!a_{k+1}x + \dots + \frac{n!}{(n-k)!}a_nx^{n-k}$$

on en déduit que

$$a_k = \frac{p^{(k)}(0)}{k!}$$

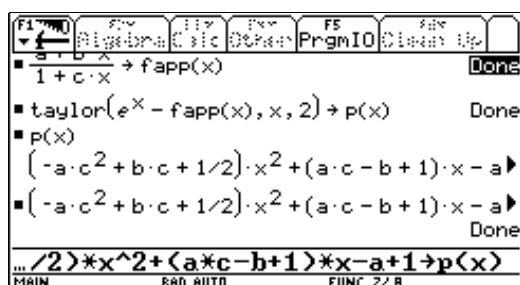
| Algorithme | Programme |
|--|---|
| Déclarer les variables locales l1 et n Initialiser n à 0 et l1 à liste vide Tant que n est inférieur ou égal au degré Calculer a_n et le stocker dans l1[n+1] Augmenter n de 1 Calculer la dérivée (n+1)ème de exp1 Fin du Tant Que Sortir l1 Fin de la fonction | <pre>coefpoly(exp1, var, d) Func local l1, n {} → l1: 0 → n While n ≤ d (exp1 var=0)/(n!) → l1[n+1] n+1 → n d(exp1, var) → exp1 Endwhile l1 EndFunc</pre> |

2 Résolution du TD à l'aide de la TI-92

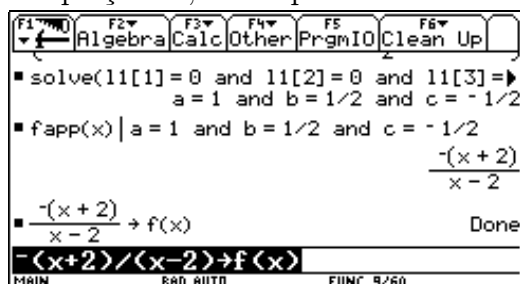
- 1) On définit $fapp(x)$.
- 2) On demande le développement de Taylor de $e^x - fapp(x)$ à l'ordre 2.
- 3) Le résultat est stocké dans $p(x)$.
- 4) On met dans 11 les coefficients de p .
- 5) On résout le système :

$$\begin{cases} 11[1] = 0 \\ 11[2] = 0 \\ 11[3] = 0 \end{cases}$$

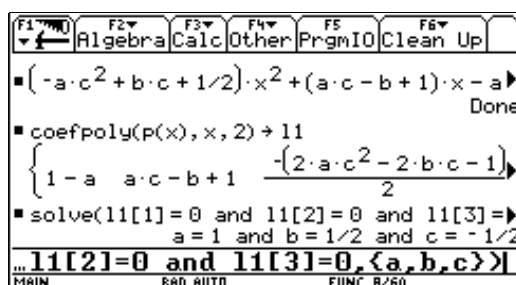
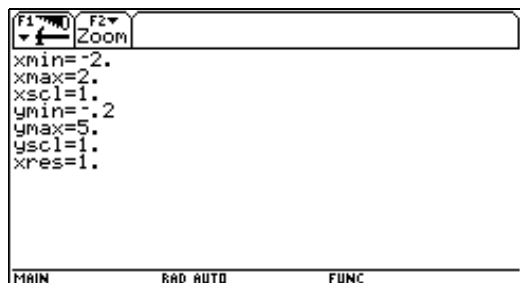
par rapport aux variables a, b et c .



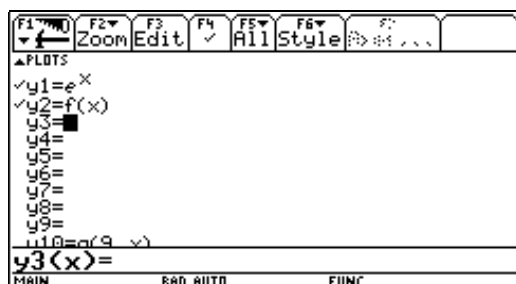
On définit alors la fonction $f = fapp$ en remplaçant a, b et c par les valeurs trouvées



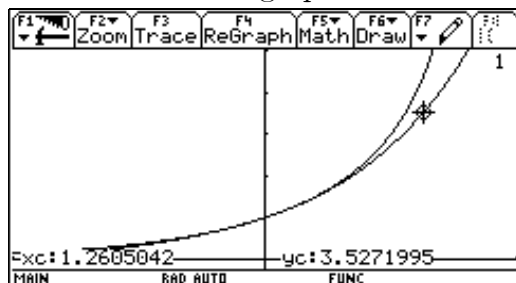
On entre la fenêtre désirée.



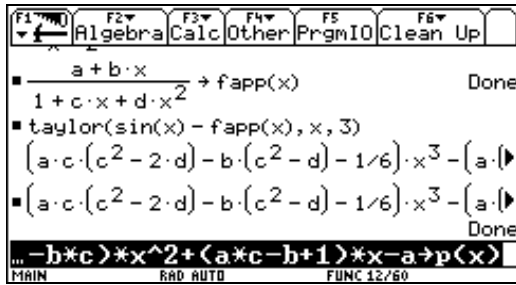
On entre les fonctions dans l'éditeur de fonctions



On trace les deux graphes.



On recommence pour la fonction $\sin(x)$ avec $p = 1$ et $q = 2$



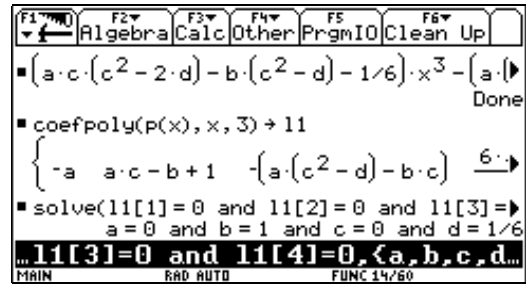
Algebra Calc Other PrgmIO Clean Up

$\frac{a+b \cdot x}{1+c \cdot x+d \cdot x^2} \rightarrow fapp(x)$ Done

taylor(sin(x) - fapp(x), x, 3)

$(a \cdot c \cdot (c^2 - 2 \cdot d) - b \cdot (c^2 - d) - 1/6) \cdot x^3 - (a \cdot b \cdot c) \cdot x^2 + (a \cdot c - b + 1) \cdot x - a + p(x)$

MAIN RAD AUTO FUNC 12/60



Algebra Calc Other PrgmIO Clean Up

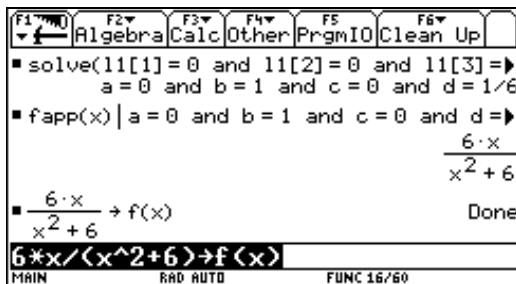
$(a \cdot c \cdot (c^2 - 2 \cdot d) - b \cdot (c^2 - d) - 1/6) \cdot x^3 - (a \cdot b \cdot c) \cdot x^2 - (-a \cdot c - b + 1) \cdot x - (a \cdot c - b + 1)$ Done

coefpoly(p(x), x, 3) \rightarrow l1

$\{-a \quad a \cdot c - b + 1 \quad -(a \cdot (c^2 - d) - b \cdot c) \quad 6 \cdot$

solve(l1[1]=0 and l1[2]=0 and l1[3]=0 and l1[4]=0, {a, b, c, d})

MAIN RAD AUTO FUNC 14/60



Algebra Calc Other PrgmIO Clean Up

solve(l1[1]=0 and l1[2]=0 and l1[3]=0 and l1[4]=0, {a, b, c, d})

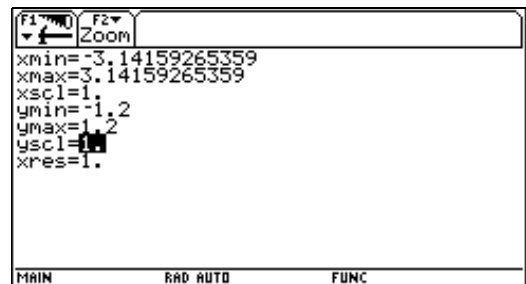
$a = 0$ and $b = 1$ and $c = 0$ and $d = 1/6$

fapp(x) | $a = 0$ and $b = 1$ and $c = 0$ and $d = 1/6$

$\frac{6 \cdot x}{x^2 + 6} \rightarrow f(x)$ Done

$6 \cdot x / (x^2 + 6) \rightarrow f(x)$

MAIN RAD AUTO FUNC 16/60



Zoom

xmin=-3.14159265359

xmax=3.14159265359

xsc1=1

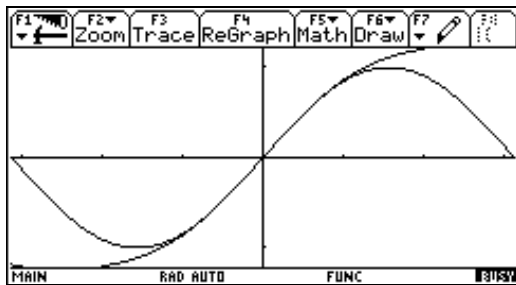
ymin=-1.2

ymax=1.2

yscl=1

xres=1

MAIN RAD AUTO FUNC



Comparaison des Padé [1/1] des fonctions $\sqrt{x+1}$ et $\frac{1}{\sqrt{x+1}}$

```

F1 Algebra F2 Calc F3 Other F4 PrgmIO F5 Clean Up F6 Up
x^2+6
a+b*x / 1+c*x -> fapp(x) Done
taylor(sqrt(1+x)-fapp(x),x,2)
(-a*c^2+b*c-1/8)*x^2+(a*c-b+1/2)*x ->
(-a*c^2+b*c-1/8)*x^2+(a*c-b+1/2)*x -> Done
... *x^2+(a*c-b+1/2)*x-a+1->p(x)
MAIN RAD AUTO FUNC 19/60
    
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```

F1 Algebra F2 Calc F3 Other F4 PrgmIO F5 Clean Up F6 Up
a+b*x / 1+c*x -> fapp1(x) Done
taylor(sqrt(1+x)-fapp1(x),x,2)
(-a*c^2+b*c-1/8)*x^2+(a*c-b+1/2)*x ->
(-a*c^2+b*c-1/8)*x^2+(a*c-b+1/2)*x -> Done
... *x^2+(a*c-b+1/2)*x-a+1->p1(x)
MAIN RAD AUTO FUNC 22/60
    
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```

F1 Algebra F2 Calc F3 Other F4 PrgmIO F5 Clean Up F6 Up
coefpoly(p(x),x,2)->11
{ 1-a a*c-b+1/2 -(8*a*c^2-8*b*c) / 8 ->
coefpoly(p1(x),x,2)->12
{ 1-a a*c-b-1/2 -(8*a*c^2-8*b*c) / 8 ->
coefpoly(p1(x),x,2)->12
MAIN RAD AUTO FUNC 24/60
    
```

```

F1 Algebra F2 Calc F3 Other F4 PrgmIO F5 Clean Up F6 Up
coefpoly(p1(x),x,2)->12
{ 1-a a*c-b-1/2 -(8*a*c^2-8*b*c) / 8 ->
solve(11[1]=0 and 11[2]=0 and 11[3]=>
a=1 and b=3/4 and c=1/4
solve(12[1]=0 and 12[2]=0 and 12[3]=>
a=1 and b=1/4 and c=3/4
... 2[1]=0 and 12[2]=0 and 12[3]...
MAIN RAD AUTO FUNC 26/60
    
```

```

F1 Algebra F2 Calc F3 Other F4 PrgmIO F5 Clean Up F6 Up
solve(12[1]=0 and 12[2]=0 and 12[3]=>
a=1 and b=1/4 and c=3/4
fapp(x)|a=1 and b=3/4 and c=1/4
3*x+4 / x+4
fapp1(x)|a=1 and b=1/4 and c=3/4
x+4 / 3*x+4
... 1(x)|a=1 and b=1/4 and c=3/4
MAIN RAD AUTO FUNC 28/60
    
```

Comparaison des Padé [2/2] des fonctions e^x et e^{-x} .

```

F1 Algebra F2 Calc F3 Other F4 PrgmIO F5 Clean Up F6
a + b · x + c · x2 → fapp1(x) Done
1 + d · x + e · x2
a + b · x + c · x2 → fapp2(x) Done
1 + d · x + e · x2
taylor(ex - fapp1(x), x, 4)
(-a · (d4 - 3 · d2 · e + e2) + b · d · (d2 - 2 · e) - c)
taylor(ex - fapp1(x), x, 4)
  
```

```

F1 Algebra F2 Calc F3 Other F4 PrgmIO F5 Clean Up F6
(-a · (d4 - 3 · d2 · e + e2) + b · d · (d2 - 2 · e) - c) Done
taylor(e-x - fapp2(x), x, 4)
(-a · (d4 - 3 · d2 · e + e2) + b · d · (d2 - 2 · e) - c)
(-a · (d4 - 3 · d2 · e + e2) + b · d · (d2 - 2 · e) - c) Done
2) · x2 + (a · d - b - 1) · x - a + 1 → p2(x)
  
```

```

F1 Algebra F2 Calc F3 Other F4 PrgmIO F5 Clean Up F6
(-a · (d4 - 3 · d2 · e + e2) + b · d · (d2 - 2 · e) - c)
d2) + b · d · (d2 - 2 · e) - c · (d2 - e) + 1/24) · x4 Done
coefpoly(p1(x), x, 4) → 11
{ 1 - a a · d - b + 1 - (2 · a · (d2 - e) - 2 · b · d) / 2 }
coefpoly(p2(x), x, 4) → 12
  
```

```

F1 Algebra F2 Calc F3 Other F4 PrgmIO F5 Clean Up F6
coefpoly(p2(x), x, 4) → 12
{ 1 - a a · d - b - 1 - (2 · a · (d2 - e) - 2 · b · d) / 2 }
solve(11[1]=0 and 11[2]=0 and 11[3]=0) ⇒
a = 1 and b = 1/2 and c = 1/12 and d = -1
solve(12[1]=0 and 12[2]=0 and 12[3]=0) ⇒
a = 1 and b = -1/2 and c = 1/12 and d = 1
12[1]=0 and 12[2]=0 and 12[3]=0
  
```

```

F1 Algebra F2 Calc F3 Other F4 PrgmIO F5 Clean Up F6
a = 1 and b = 1/2 and c = 1/12 and d = -1
solve(12[1]=0 and 12[2]=0 and 12[3]=0) ⇒
a = 1 and b = -1/2 and c = 1/12 and d = 1
11[1] 1 - a
11[2] a · d - b + 1
11[3] -(2 · a · (d2 - e) - 2 · b · d + 2 · c - 1) / 2
11[3]
  
```

```

F1 Algebra F2 Calc F3 Other F4 PrgmIO F5 Clean Up F6
fapp1(x) | a = 1 and b = 1/2 and c = 1/12 ⇒
x2 + 6 · x + 12
x2 - 6 · x + 12
fapp2(x) | a = 1 and b = -1/2 and c = 1/12 ⇒
x2 - 6 · x + 12
x2 + 6 · x + 12
c = 1/12 and d = 1/2 and e = 1/12
  
```

Les calculs deviennent si l'on procède par l'intermédiaire du produit :

| | | | | | |
|---|------|-------|--------|----------|----|
| F1 | F2 | F3 | F4 | F5 | F6 |
| Algebra | Calc | Other | PrgmIO | Clean Up | |
| $x^2 + 6 \cdot x + 12$ | | | | | |
| $\blacksquare \text{taylor}(e^x \cdot (1 + d \cdot x + e \cdot x^2) - (a + b \cdot x + c \cdot x^2), x, 4) \rightarrow$ $\frac{(4 \cdot d + 12 \cdot e + 1) \cdot x^4}{24} + \frac{(3 \cdot d + 6 \cdot e + 1) \cdot x^3}{6} \rightarrow$ | | | | | |
| $\blacksquare \frac{(4 \cdot d + 12 \cdot e + 1) \cdot x^4}{24} + \frac{(3 \cdot d + 6 \cdot e + 1) \cdot x^3}{6} \rightarrow$ | | | | | |
| Done | | | | | |
| $\blacksquare \dots 1 \cdot x^2 / 2 + (-b + d + 1) \cdot x - a + 1 \rightarrow p(x)$ | | | | | |
| MAIN | RAD | AUTO | FUNC | 45/60 | |

| | | | | | |
|---|------|-------|--------|----------|----|
| F1 | F2 | F3 | F4 | F5 | F6 |
| Algebra | Calc | Other | PrgmIO | Clean Up | |
| $x^2 - 6 \cdot x + 12$ | | | | | |
| $\blacksquare \frac{(4 \cdot d + 12 \cdot e + 1) \cdot x^4}{24} + \frac{(3 \cdot d + 6 \cdot e + 1) \cdot x^3}{6} \rightarrow$ | | | | | |
| Done | | | | | |
| $\blacksquare \text{coefpoly}(p(x), x, 4) \rightarrow 11$ $\left\{ \begin{array}{l} 1 - a \quad -b + d + 1 \quad \frac{-(2 \cdot c - 2 \cdot d - 2 \cdot e - 1)}{2} \end{array} \right\}$ | | | | | |
| $\blacksquare \text{solve}(11[1]=0 \text{ and } 11[2]=0 \text{ and } 11[3]=0) \rightarrow$ $a = 1 \text{ and } b = 1/2 \text{ and } c = 1/12 \text{ and } d = -1$ | | | | | |
| $\blacksquare \dots 11[1]=0 \text{ and } 11[2]=0 \text{ and } 11[3]=0$ | | | | | |
| MAIN | RAD | AUTO | FUNC | 47/60 | |

| | | | | | |
|---|------|-------|--------|----------|----|
| F1 | F2 | F3 | F4 | F5 | F6 |
| Algebra | Calc | Other | PrgmIO | Clean Up | |
| $\blacksquare \frac{a + b \cdot x + c \cdot x^2}{1 + d \cdot x + e \cdot x^2} \mid a = 1 \text{ and } b = 1/2 \text{ and } c =$ | | | | | |
| Done | | | | | |
| $\blacksquare \frac{x^2 + 6 \cdot x + 12}{x^2 - 6 \cdot x + 12} \rightarrow f(x)$ | | | | | |
| $\blacksquare \dots \frac{x^2 + 6 \cdot x + 12}{x^2 - 6 \cdot x + 12} \rightarrow f(x)$ | | | | | |
| MAIN | RAD | AUTO | FUNC | 49/60 | |

| | | | | | |
|--|------|-------|--------|----------|----|
| F1 | F2 | F3 | F4 | F5 | F6 |
| Algebra | Calc | Other | PrgmIO | Clean Up | |
| $x^2 - 6 \cdot x + 12$ | | | | | |
| $\blacksquare \text{taylor}(\tan(x) \cdot (1 + e \cdot x^2) - (b \cdot x + c \cdot x^3), x, 5) \rightarrow$ $\frac{(5 \cdot e + 2) \cdot x^5}{15} - \frac{(3 \cdot c - 3 \cdot e - 1) \cdot x^3}{3} - (b - 1) \cdot x$ | | | | | |
| $\blacksquare \frac{(5 \cdot e + 2) \cdot x^5}{15} - \frac{(3 \cdot c - 3 \cdot e - 1) \cdot x^3}{3} - (b - 1) \cdot x \rightarrow$ | | | | | |
| Done | | | | | |
| $\blacksquare \dots c - 3 \cdot e - 1 \cdot x^3 / 3 - (b - 1) \cdot x \rightarrow p(x)$ | | | | | |
| MAIN | RAD | AUTO | FUNC | 51/60 | |

Recherche du Pade [3/3] de $\tan(x)$:

| | | | | | |
|--|------|-------|--------|----------|----|
| F1 | F2 | F3 | F4 | F5 | F6 |
| Algebra | Calc | Other | PrgmIO | Clean Up | |
| $x^2 - 6 \cdot x + 12$ | | | | | |
| $\blacksquare \text{taylor}(\tan(x) \cdot (1 + e \cdot x^2) - (b \cdot x + c \cdot x^3), x, 5) \rightarrow$ $\frac{(5 \cdot e + 2) \cdot x^5}{15} - \frac{(3 \cdot c - 3 \cdot e - 1) \cdot x^3}{3} - (b - 1) \cdot x$ | | | | | |
| $\blacksquare \frac{(5 \cdot e + 2) \cdot x^5}{15} - \frac{(3 \cdot c - 3 \cdot e - 1) \cdot x^3}{3} - (b - 1) \cdot x \rightarrow$ | | | | | |
| Done | | | | | |
| $\blacksquare \dots c - 3 \cdot e - 1 \cdot x^3 / 3 - (b - 1) \cdot x \rightarrow p(x)$ | | | | | |
| MAIN | RAD | AUTO | FUNC | 51/60 | |

| | | | | | |
|---|------|-------|--------|----------|----|
| F1 | F2 | F3 | F4 | F5 | F6 |
| Algebra | Calc | Other | PrgmIO | Clean Up | |
| $x^2 - 6 \cdot x + 12$ | | | | | |
| $\blacksquare \frac{(5 \cdot e + 2) \cdot x^5}{15} - \frac{(3 \cdot c - 3 \cdot e - 1) \cdot x^3}{3} - (b - 1) \cdot x \rightarrow$ | | | | | |
| Done | | | | | |
| $\blacksquare \text{coefpoly}(p(x), x, 5) \rightarrow 11$ $\left\{ \begin{array}{l} 0 \quad 1 - b \quad 0 \quad \frac{-(3 \cdot c - 3 \cdot e - 1)}{3} \quad 0 \quad \frac{5 \cdot e}{15} \end{array} \right\}$ | | | | | |
| $\blacksquare \text{solve}(11[2]=0 \text{ and } 11[4]=0 \text{ and } 11[6]=0) \rightarrow$ $b = 1 \text{ and } c = -1/15 \text{ and } e = -2/5$ | | | | | |
| $\blacksquare \dots 11[4]=0 \text{ and } 11[6]=0, \langle c, e, b \rangle$ | | | | | |
| MAIN | RAD | AUTO | FUNC | 53/60 | |

| | | | | | |
|---|------|-------|--------|----------|----|
| F1 | F2 | F3 | F4 | F5 | F6 |
| Algebra | Calc | Other | PrgmIO | Clean Up | |
| $\blacksquare \frac{b \cdot x + c \cdot x^2}{1 + e \cdot x^2} \mid b = 1 \text{ and } c = -1/15 \text{ and } e =$ | | | | | |
| Done | | | | | |
| $\blacksquare \frac{x \cdot (x^2 - 15)}{3 \cdot (2 \cdot x^2 - 5)} \rightarrow f(x)$ | | | | | |
| $\blacksquare \dots \frac{x \cdot (x^2 - 15)}{3 \cdot (2 \cdot x^2 - 5)} \rightarrow f(x)$ | | | | | |
| MAIN | RAD | AUTO | FUNC | 55/60 | |

| | |
|---|------|
| F1 | F2 |
| Zoom | |
| $\text{Sur }]-\frac{\pi}{2}; \frac{\pi}{2}[$ | |
| $x_{\min} = -1.57079632679$ $x_{\max} = 1.57079632679$ $x_{\text{sc1}} = 1.$ $y_{\min} = -10.$ $y_{\max} = 10.$ $y_{\text{sc1}} = 1.$ $x_{\text{res}} = 1.$ | |
| MAIN | FUNC |

